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Title: Symmetric Nonnegative Trifactorization Rank

Abstract: Factorizations of matrices where the factors are required to be entry-wise nonnegative provide a powerful tool in analysing nonnegative data. In this talk we will consider Symmetric Nonnegative Matrix Trifactorization (SN-Trifactorization), a factorization of nonnegative symmetric matrices that takes into account symmetry, nonnegativity and low rank of a matrix.

SN-Trifactorization is a factorization of an n x n (entry-wise) nonnegative symmetric matrix A of the form BCB^T, where C is a k x k symmetric matrix, and both B and C are required to be nonnegative. The SNT-rank of A is the minimal k for which such factorization exists. The zero-nonzero pattern of A poses constrains on the zero-nonzero pattern of B and C satisfying $A = BCB^T$. Those constraints can be studied through the SNT-rank of simple graphs that allow loops, defined to be the minimal possible SNT-rank of all symmetric nonnegative matrices whose zero-nonzero pattern is prescribed by a given graph.

After introducing the SNT-rank of nonnegative symmetric matrices and exploring its basic properties, we will turn our discussion to the SNT-rank of graphs. We will define set-join covers of graphs, and show that finding the SNT-rank of a graph G is equivalent to finding the minimal order of a set-join cover of G. Using this insight we will develop basic properties of the SNT-rank for graphs and compute it for trees and cycles without loops. We will show the equivalence between the SNT-rank for complete graphs and the Katona problem, and discuss uniqueness of patterns of matrices in the factorization.