Title: A solution to the extreme point problem in Lipschitz-free spaces

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Abstract:

Lipschitz-free (hereafter free) spaces, also known as Arens-Eells spaces or transportation cost spaces, are Banach spaces that can be found at the interface between functional analysis, metric geometry and optimal transport theory. The free space norm naturally extends the 1-Wasserstein (or Kantorovich-Rubinstein) metric on the space of probability measures having finite first moment. Free spaces (and their duals) are arguably the canonical way to express metric spaces in functional analytic terms, analogously to how compact Hausdorff spaces and measure spaces can be expressed using C(K)-spaces and L_p-spaces, respectively. While free spaces are easy to define using so-called `elementary molecules', their structure remains relatively poorly understood and many basic questions remain open. When studying a Banach space, one of the key tasks is to identify the extreme points of its (closed) unit ball. We show that all extreme points of the unit ball of a free space are elementary molecules, solving a problem that can be traced to the mid-90s. The proof relies on a `Choquet theory of free spaces', elements of which I will present in the talk. This is partially joint work with R. J. Aliaga (Universitat Politècnica de Valencia) and E. Pernecká (Czech Technical University, Prague).