Title: Jacobson isopotency

Abstract:

Given a natural number k, let us say that a ring R satisfies the Jacobson k identity if $x^{k+1} = x$ for all x in R. Any such identity implies that the ring is commutative; this rather well-known fact is proved as an application of Jacobson's structure theory.

We say that natural numbers m and n are isopotent if the Jacobson m- and n identities are logically equivalent. This sets up an equivalence relation on N and we call the smallest element of each equivalence class the stem. For instance, each element of {2, 3, 4, 6, 7, 8, 14} is a stem, whereas no element of {5, 9, 11, 13, 17, 19} is a stem. There are simple number theoretic conditions that determine whether n and m are isopotent and whether n is a stem. However, we aim to demonstrate that isopotency classes and stems are still worthy of further investigation.

This talk is inspired by ring theory but is mainly number theoretic. We spoke on this topic in a colloquium here a year ago, but this is an updated report.